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International Journal of Heat and Mass Transfer 49 (2006) 1851–1856

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to suction and to a transverse magnetic field

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Received 1 March 2005; received in revised form 16 November 2005
Available online 23 January 2006

Abstract

An analysis is performed for flow and heat transfer of a steady laminar boundary-layer flow of an electrically conducting fluid of second grade subject to suction and to a transverse uniform magnetic field past a semi-infinite stretching sheet. The governing partial differential equations are converted into ordinary differential equations by a similarity transformation and an analytical solution for this flow is utilized. The effects of viscous dissipation and work due to deformation are considered in the energy equation and the variations of dimensionless surface temperature and dimensionless surface temperature gradient with various parameters are graphed and tabulated. Two cases are studied, namely, (i) the sheet with constant surface temperature (CST case) and (ii) the sheet with prescribed surface temperature (PST case).

Keywords: MHD flow and heat transfer; Suction; Second grade fluid; Stretching sheet

1. Introduction

Boundary layer behaviour over a moving continuous solid surface is an important type of flow occurring in several engineering processes. Since the pioneering work of Sakiadis [1,2], various aspects of the problem have been investigated by many authors. Crane [3], Vleggaar [4] and Gupta and Gupta [5] have analyzed the stretching problem with constant surface temperature while Soundalgekar and Ramana Murty [6] investigated the constant surface velocity case with power-law temperature variation. This flow was examined by Siddappa and Khapate [7] for a special class of non-Newtonian fluids known as second-order fluids which are viscoelastic in nature.

Rajagopal et al. [8] independently examined the same flow as in [7] and obtained similarity solutions of the boundary layer equations numerically for the case of small viscoelastic parameter k_1. It is shown that skin-friction decreases with increase in k_1. Dandapat and Gupta [9] examined the same problem with heat transfer. In [9], an exact analytical solution of the non-linear equation governing this self-similar flow which is consistent with the numerical results in [8] is given and the solutions for the temperature for various values of k_1 are presented. Later, Cortell [10] extended the work of Dandapat and Gupta [9] to study the heat transfer in an incompressible second-order fluid caused by a stretching sheet with a view to examining the influence of the viscoelastic parameter on temperature distributions. It is found that temperature distribution depends on k_1, in accordance with the results in [9]. Numerical solutions for the flow of a fluid of grade three past an infinite porous flat plate subject to suction at the plate are to be found in Rajagopal et al. [11] and in Cortell [12]. Hayat et al. [13] studied the flow of a third-grade fluid over a wall with suction or blowing and

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0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2005.11.013

Gupta et al. [14] investigated the steady flow of a power-law fluid past an infinite porous flat plate subject to suction or blowing with heat transfer. Arbitrary injection/suction in a power-law fluid is analyzed in [15]. Flow and heat transfer characteristics were investigated in [16] for a visco-elastic fluid over a stretching sheet with power-law surface temperature and in [17] with a non-linearly stretching sheet. Very recently, Vajravelu and Rollings [18] assumed additional effects such as the flow in an electrically conducting fluid permeated by a transverse uniform magnetic field with uniform suction at the surface, however, heat transfer in such flow was not studied.

Furthermore, they augmented the missing boundary condition and used a proper sign for the normal stress modulus (i.e. $\alpha_1 \geqslant 0$). In the present paper the same model as in [18] is used and we investigate both momentum transfer and heat transfer boundary-layer problems. The fluid is at rest and the motion is created by the surface whose velocity varies linearly with the distance x from a fixed point and the sheet is held at a temperature $T_w(x)$ higher than the temperature T_∞ of the ambient fluid. Further, an exact analytical solution of the above-mentioned flow is utilized.

In Section 2, we shall consider the mathematical analysis of the flow and some exact solutions; and in Section 3 we shall examine the thermal problem when both dissipative heat and work due to deformation are included in the energy equation; furthermore, the influence on the numerical results of these additional effects will also be discussed.

2. Flow analysis

An incompressible homogeneous second grade fluid has a constitutive equation given by [19]

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2. \tag{1}$$

Here $\mathbf{T}$ is the stress tensor, p the pressure, μ the coefficient of viscosity, α_1, α_2 are material constants and $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as

$$\mathbf{A}_1 = (\mathrm{grad}\,\mathbf{v}) + (\mathrm{grad}\,\mathbf{v})^{\mathrm{T}}, \tag{2}$$

$$\mathbf{A}_2 = \mathrm{d}/\mathrm{d}t\mathbf{A}_1 + \mathbf{A}_1 \cdot \mathrm{grad}\,\mathbf{v} + (\mathrm{grad}\,\mathbf{v})^{\mathrm{T}} \cdot \mathbf{A}_1. \tag{3}$$

Here $\mathbf{v}$ denotes the velocity field and $\mathrm{d}/\mathrm{d}t$ is the material time derivative. Some assumptions concerning the sign of α_1 in the model (1) will be necessary. For thermodynamic reasons (see [20]), the material parameter α_1 must be positive. Furthermore, a thorough discussion of these issues can be found in the critical review of Dunn and Rajagopal [21]. We do not intend to discuss here the sign of α_1, but to study per se the influence of the viscoelastic parameter λ_1, the magnetic parameter M and the suction parameter R on flow, heat-transfer characteristics and temperature distributions. In our analysis we assume that the fluid is thermodynamically compatible. Let us suppose a steady, laminar and two-dimensional flow of an incompressible, electrically conducting second grade fluid subject to transverse magnetic field past a flat sheet coinciding with the plane $y = 0$ and the flow being confined to $y > 0$. Two equal and opposite forces are introduced along the x-axis so that the surface is stretched keeping the origin fixed. We take x-axis along the surface, the y-axis being normal to it and u and v are the fluid tangential velocity and normal velocity, respectively. Thus, for the problem under consideration the equations of the laminar boundary layer are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho}u + \frac{\alpha_1}{\rho}\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right], \tag{5}$$

where ν is the kinematic viscosity, ρ is the density, σ_0 is the electric conductivity, B_0 is the uniform magnetic field along the y-axis and α_1 is the material constant.

The boundary conditions for the velocity field are

$$u = cx, \quad v = -v_0 \quad \text{at } y = 0,\ c > 0, \tag{6}$$

$$u \to 0, \quad \frac{\partial u}{\partial y} \to 0 \quad \text{as } y \to \infty, \tag{7}$$

where c is the stretching rate.

Defining new variables

$$u = cxf'(\eta), \quad v = -(c \cdot \nu)^{1/2} f(\eta) \tag{8}$$

where

$$\eta = \left(\frac{c}{\nu}\right)^{1/2} y. \tag{9}$$

With these changes of variables, Eq. (4) is identically satisfied and substituting in (5) gives

$$(f')^2 - ff'' + Mf' = f''' + \lambda_1\lfloor 2f'f''' - (f'')^2 - ff^{iv}\rfloor, \tag{10}$$

with boundary conditions

$$f' = 1, \quad f = R \quad \text{at } \eta = 0, \tag{11}$$

$$f' \to 0, \quad f'' \to 0 \quad \text{as } \eta \to \infty. \tag{12}$$

Here prime denotes differentiation with respect to η, $\lambda_1 = \frac{\alpha_1 c}{\rho\nu}$ is the viscoelastic parameter, $M = \frac{\sigma_0 B_0^2}{\rho c}$ is the magnetic parameter and $R = \frac{v_0}{(c\nu)^{1/2}}$ is the suction parameter.

It is interesting to note that the problem (10)–(12) has a solution of the form [18]

$$f(\eta) = R + \frac{1 - \exp(-b\eta)}{b}, \tag{13}$$

where $b(>0)$ satisfies the following cubic equation:

$$\lambda_1 R b^3 + (1 + \lambda_1)b^2 - Rb - M - 1 = 0, \tag{14}$$

for arbitrary and positive values of R, λ_1 and M.

Thus, from (13) and (14), we get a simple exact analytical solution of the problem (10)–(12) and we use in heat transfer analysis this solution for the function f.

For details about Eq. (14), the reader is referred to Ref. [18]. A detailed discussion regarding to get some idea of how b depends on the parameters R, λ_1 and M was carried out in that paper by using perturbation techniques. For small R and λ_1 explicit expressions for b can be found, however, for arbitrary values of R, λ_1 and M it is best to find the value of b numerically [18].

The velocity components are

$$u = cx\exp(-b\eta);$$
$$v = -(cv)^{1/2}\left(R + \frac{1-\exp(-b\eta)}{b}\right). \tag{15}$$

On the other hand, we obtain from (13) that the skin friction parameter $-f''(0)$ is equal to b. The shear stress at a point on the surface is

$$\tau_0 = -\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu x b c\sqrt{\frac{c}{v}}, \tag{16}$$

where μ is the viscosity. The non-dimensional form of the shear stress is

$$\tau = \frac{\tau_0}{c^2x^2\rho} \tag{17}$$

and we obtain from (16)

$$\tau = \frac{b}{x}\sqrt{\frac{v}{c}}. \tag{18}$$

From Eq. (8) it can be seen that the horizontal velocity profiles are related with $f'(\eta) = \exp(-b\eta)$. It is observed from this equation that the velocity component u decreases in the boundary layer with increase of η. Further, the effects on both skin-friction parameter b and shear stress in the boundary of the parameters M, R and λ_1 can be analyzed from Eqs. (14)–(18).

3. Heat transfer analysis

By using boundary layer approximations, and taking into account both viscous dissipation and work due to deformation the equation of energy for temperature T is given by

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{v}{c_P}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\alpha_1}{\rho c_P}\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\right], \tag{19}$$

where α is the thermal diffusivity and c_P is the specific heat of a fluid at constant pressure.

3.1. Constant surface temperature (CST case)

In this circumstance, the boundary conditions are

$$T = T_w \text{ at } y = 0, \quad T \to T_\infty \text{ as } y \to \infty, \tag{20}$$

where T_w and T_∞ are constants.

Defining the non-dimensional temperature $\theta(\eta)$ and the Prandtl number σ as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \sigma = \frac{v}{\alpha} \tag{21}$$

and using (8) and (9), we find from (19)

$$\theta'' + \sigma f\theta' = -\sigma E_c\lfloor (f'')^2 + \lambda_1 f''(f'f'' - ff''')\rfloor \tag{22}$$

with the boundary conditions

$$\theta(0) = 1, \quad \theta(\infty) \to 0. \tag{23}$$

Here, $E_c = \frac{b^2x^2}{c_P(T_w - T_\infty)}$ represents the appropriate form of the Eckert number for this problem (see [10]). It is worth mentioning that the x-coordinate can not be eliminated from Eq. (22), whereby, the temperature profiles always depend on x. It is clear from Eq. (22) that its right-hand side vanishes in the absence of all the effects and that all solutions are then of the similar type. When those effects are neglected, we obtain the simpler equation

$$\theta'' + \sigma f\theta' = 0. \tag{24}$$

Using numerical methods of integration and disregarding temporarily the second condition (23), a family of solutions of (24) can be obtained for arbitrarily chosen values of $\left(\frac{d\theta}{d\eta}\right)_{\eta=0} = \theta'(0) \leqslant 0$. Tentatively we assume that a special value of $|\theta'(0)|$ yields a solution for which θ vanishes at a certain $\eta = \eta_\infty$ and satisfies the additional condition

$$\frac{d\theta}{d\eta} = 0, \quad \theta = 0 \text{ at } \eta = \eta_\infty. \tag{25}$$

We guess $\theta'(0)$ and integrate Eq. (24) and first condition (23) as an initial value problem by the Runge–Kutta method of fourth order with the additional condition (25). In the present study, the equivalent step size $\Delta\eta = 0.02$ is used to obtain the numerical solution. It is worth mentioning that, for each numerical solution, the η_∞ value depends on the non-dimensional parameters M, R, λ_1, σ and E_c. We follow an iterative procedure which is stopped to give the temperature and temperature-gradient distributions when (25) is reached and the error in the value of $|\theta'(0)|$ becomes less than 10^{-4}. We have also studied the effect of the step size $\Delta\eta$ on some numerical solutions and for $0.01 < \Delta\eta < 0.02$ the results here are independent of $\Delta\eta$ almost up to the fourth decimal place. In this manner, problem (24) and (23) was solved. The results of the numerical solutions for various values of λ_1 with $\sigma = 0.7$, $M = 1$ and $R = 5$ are shown in Table 1. It is seen from Table 1 that for a given position η, $\theta(\eta)$ decreases as the viscoelastic parameter λ_1 increases.

On the other hand, Fig. 1 shows the effect of Prandtl number σ on temperature and temperature-gradient profiles. Fig. 1 indicates that for a given location η, $\theta(\eta)$ decreases as the σ increases, resulting in a decrease of the thermal boundary layer thickness.

3.2. Prescribed surface temperature (PST case)

Here, the boundary conditions are

$$\begin{aligned} &T = T_w(= T_\infty + A\cdot x^s) \quad \text{at } y = 0, \\ &T \to T_\infty \quad \text{as } y \to \infty, \end{aligned} \tag{26}$$

where s is the wall temperature parameter.

Table 1
Values of $\theta(\eta)$ and $\theta'(\eta)$ in the CST case with $\sigma = 0.7$; $M = 1$ and $R = 5$

R	M	σ	λ_1	b	η	θ	$-\theta'$
5	1	0.7	0.1	2.480883	0.0	1.0	3.613778
					0.1	0.695396	2.538390
					0.2	0.482013	1.773252
					0.5	0.158438	0.591277
					1.0	0.024291	0.091342
					2.0	0.000604	0.002098
					4.0	4.93×10^{-5}	1.1×10^{-6}
			0.5	1.339356	0.0	1.0	3.638282
					0.1	0.693339	2.555291
					0.2	0.478596	1.783699
					0.5	0.154073	0.589009
					1.0	0.022206	0.086890
					2.0	0.000462	0.001678
					4.0	4.2×10^{-5}	0.6×10^{-6}
			1.7	0.794896	0.0	1.0	3.653577
					0.1	0.692054	2.565879
					0.2	0.476453	1.790379
					0.5	0.151255	0.587871
					1.0	0.020790	0.083965
					2.0	0.000373	0.001383
					4.0	4.6×10^{-5}	0.3×10^{-6}

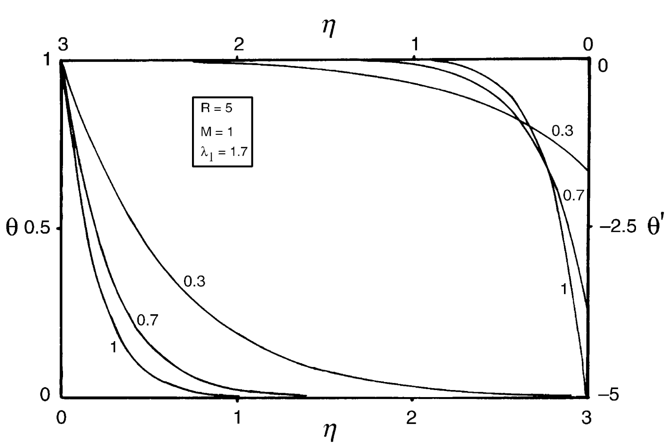

Fig. 1. Temperature and temperature-gradient profiles in the CST case for several values of σ with $R = 5$; $M = 1$ and $\lambda_1 = 1.7$ (σ values are indicated on the curves).

Using Eqs. (8) and (9), (19) and conditions (26) can be written as

$$\theta'' + \sigma f\theta' - s\sigma f'\theta = -\sigma E_c x^{2-s} \cdot \lfloor (f'')^2 + \lambda_1 f''(f'f'' - ff''') \rfloor, \quad (27)$$

$$\theta(0) = 1, \quad \theta(\infty) \to 0, \quad (28)$$

where $E_c = c^2/Ac_P$.

If $s = 2$, we find from (27)

$$\theta'' + \sigma f\theta' - 2\sigma f'\theta = -\sigma E_c \lfloor (f'')^2 + \lambda_1 f''(f'f'' - ff''') \rfloor. \quad (29)$$

It is clear from Eq. (29) that all solutions are then of the similar type. If we do not take into account neither the viscous dissipation nor heat due to elastic deformation, we obtain from (29) the simpler equation

$$\theta'' + \sigma f\theta' - 2\sigma f'\theta = 0. \quad (30)$$

On the other hand, for negligible effects, we find from Eq. (27)

$$\theta'' + \sigma f\theta' - s\sigma f'\theta = 0, \quad (31)$$

where s is now arbitrary.

The problem (30) and (28) was solved for several values of λ_1 with $\sigma = 0.7$, $R = 5$ and $M = 1$. The computational results are listed in Table 2. It can be seen from Table 2 that the temperature at a point decreases with the viscoelastic parameter λ_1.

If the effects of viscous dissipation and work due to deformation are considered, we look at the problem (29) and (28). This problem was also solved for several values of λ_1 with $\sigma = 0.7$, $R = 5$, $M = 1$ and $E_c = 0.02$. The results are shown in Table 3.

It can be seen from Table 3 that in this case and for a given position η, $\theta(\eta)$ decrease as the viscoelastic parameter λ_1 increase.

On the other hand, we present in Table 4 numerical results for the same last thermal problem, but without considering the work due to deformation. Obviously, we can analyze this effect on temperature and temperature-gradient profiles by comparing numerical results in Tables 3 and 4.

Finally, a selected set of numerical solutions is plotted in Figs. 2 and 3. For that set of non-dimensional parameters we can observe that the effect of increasing values of R is to decrease the temperature distribution, whereas an opposite behaviour can be seen for the magnetic parameter M.

Table 2
Values of $\theta(\eta)$ and $\theta'(\eta)$ in the PST case with $\sigma = 0.7$; $M = 1$ and $R = 5$

E_c	R	M	σ	λ_1	b	η	θ	$-\theta'$
0	5	1	0.7	0.1	2.480883	0.0	1.0	3.833174
						0.1	0.681989	2.606685
						0.2	0.465567	1.775401
						0.5	0.148725	0.564589
						1.0	0.022386	0.084589
						2.0	0.000555	0.001921
						4.0	4.78×10^{-5}	1.0×10^{-6}
				0.7	1.163810	0.0	1.0	3.910973
						0.1	0.675641	2.655720
						0.2	0.455638	1.799041
						0.5	0.138435	0.552529
						1.0	0.018596	0.074987
						2.0	0.000353	0.001230
						4.0	3.5×10^{-5}	0.2×10^{-6}
				1.7	0.794896	0.0	1.0	3.937998
						0.1	0.673423	2.673074
						0.2	0.452126	1.807886
						0.5	0.134641	0.548574
						1.0	0.017159	0.071300
						2.0	0.000325	0.001053
						4.0	8.5×10^{-5}	1.2×10^{-6}

Table 3
Values of $\theta(\eta)$ and $\theta'(\eta)$ in the PST case with $\sigma = 0.7$; $M = 1$; $R = 5$ and $E_c = 0.02$ when the work due to deformation is taken into account

E_c	R	M	σ	λ_1	b	η	θ	$-\theta'$
0.02	5	1	0.7	0.1	2.480883	0.0	1.0	3.814765
						0.1	0.683185	2.599881
						0.2	0.467122	1.774298
						0.5	0.149810	0.566984
						1.0	0.022673	0.085391
						2.0	0.000603	0.001948
						4.0	8.9×10^{-5}	1.0×10^{-6}
				0.7	1.163810	0.0	1.0	3.898615
						0.1	0.676552	2.649388
						0.2	0.456981	1.796406
						0.5	0.139792	0.553891
						1.0	0.019211	0.076174
						2.0	0.000423	0.001463
						4.0	3.4×10^{-5}	1.9×10^{-6}
				1.7	0.794896	0.0	1.0	3.925575
						0.1	0.674371	2.666139
						0.2	0.453577	1.804491
						0.5	0.136294	0.549504
						1.0	0.018098	0.072678
						2.0	0.000471	0.001410
						4.0	1.1×10^{-5}	1.5×10^{-5}

Table 4
Values of $\theta(\eta)$ and $\theta'(\eta)$ in the PST case with $\sigma = 0.7$; $M = 1$; $R = 5$ and $E_c = 0.02$ when the work due to deformation is not taken into account

E_c	R	M	σ	λ_1	b	η	θ	$-\theta'$
0.02	5	1	0.7	0.1	2.480883	0.0	1.0	3.816686
						0.1	0.683055	2.600675
						0.2	0.466945	1.774523
						0.5	0.149652	0.566813
						1.0	0.022576	0.085328
						2.0	0.000524	0.001946
						4.0	1.0×10^{-5}	1.0×10^{-6}
				0.7	1.163810	0.0	1.0	3.903775
						0.1	0.676171	2.652046
						0.2	0.456418	1.797527
						0.5	0.139216	0.553345
						1.0	0.018936	0.075690
						2.0	0.000370	0.001397
						4.0	1.0×10^{-5}	1.3×10^{-6}
				1.7	0.794896	0.0	1.0	3.933468
						0.1	0.673768	2.670556
						0.2	0.452653	1.806665
						0.5	0.135236	0.548916
						1.0	0.017485	0.071816
						2.0	0.000354	0.001187
						4.0	3.0×10^{-5}	5.2×10^{-6}

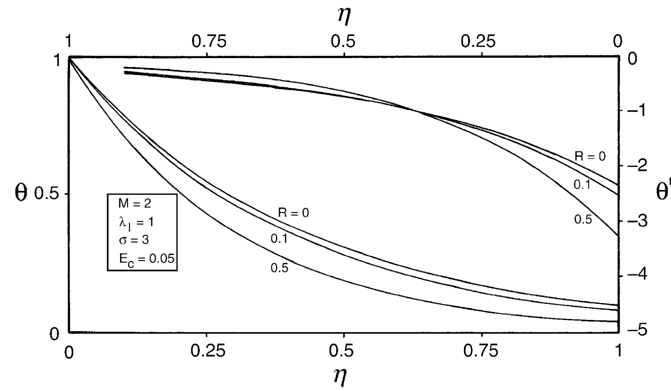

Fig. 2. Temperature and temperature-gradient profiles in the PST case for several values of R when $M = 2$; $\lambda_1 = 1$; $\sigma = 3$ and $E_c = 0.05$.

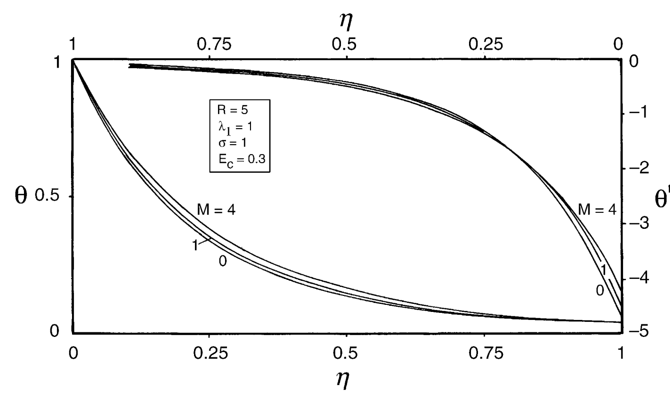

Fig. 3. Temperature and temperature-gradient profiles in the PST case for several values of M when $R = 5$; $\lambda_1 = 1$; $\sigma = 1$ and $E_c = 0.3$.

4. Discussions and conclusions

The flow and heat transfer in a laminar flow of an incompressible and electrically conducting second grade fluid subject to suction and to a transverse uniform magnetic field past a stretching sheet have been examined. The energy equation includes both the viscous dissipation and work due to deformation. A parameter of interest for the present study is the viscoelastic parameter λ_1 which is related to α_1. The values of f' and f are related to the velocity components u and v through Eqs. (14) and (15). From these equations it can be studied the behaviour of u and v with changes in λ_1. The equations for the heat transfer analysis were solved by the Runge–Kutta method of fourth order and two different cases have been analyzed:

Case 1 Constant surface temperature (*CST case*). It is seen from Table 1 that for $R = 5$; $M = 1$; $\sigma = 0.7$ and for a given position η, $\theta(\eta)$ decreases as the viscoelastic parameter λ_1 increases and the dimensionless heat transfer coefficient $[-\theta'(0)]$ increases with increase in λ_1.

Case 2 Prescribed surface temperature (*PST case*). Similarity solutions can be obtained in this case and our study includes both viscous dissipation and work due to deformation. Furthermore, the effect on both temperature and temperature-gradient profiles when the contribution of heat due to elastic deformation is taken into account in the energy equation can be analyzed by comparing numerical results given in Tables 3 and 4. Note that in these Tables we use the same values of M, R, λ_1, σ and E_c.

It is concluded from the analysis that, in general, the combined effect of increasing values of λ_1, σ and R is to increase the numerical value of wall temperature gradient $|\theta'(0)|$; consequently, more heat is carried out of the sheet, resulting in a decrease of the thermal boundary layer thickness and hence increasing the heat transfer rate, whereas an opposite behaviour can be found for E_c and M. Also, the presence of the work done by deformation's effect in the energy equation yields an augment in the fluid's temperature.

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